

# Key Concept Circle and Angle Relationships

Vertex of Angle	Model(s)	Angle Measure
on the circle		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

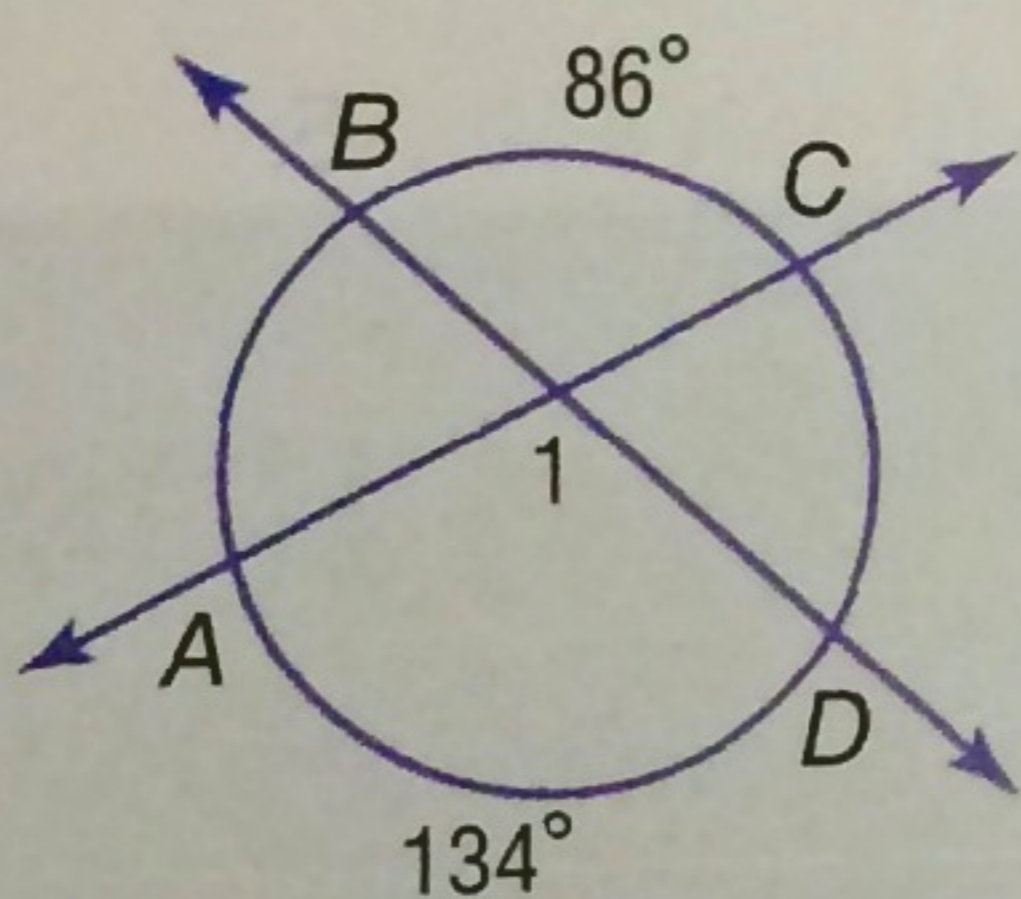
## Check Your Understanding

= Step-by-Step Solutions begin on page R14.

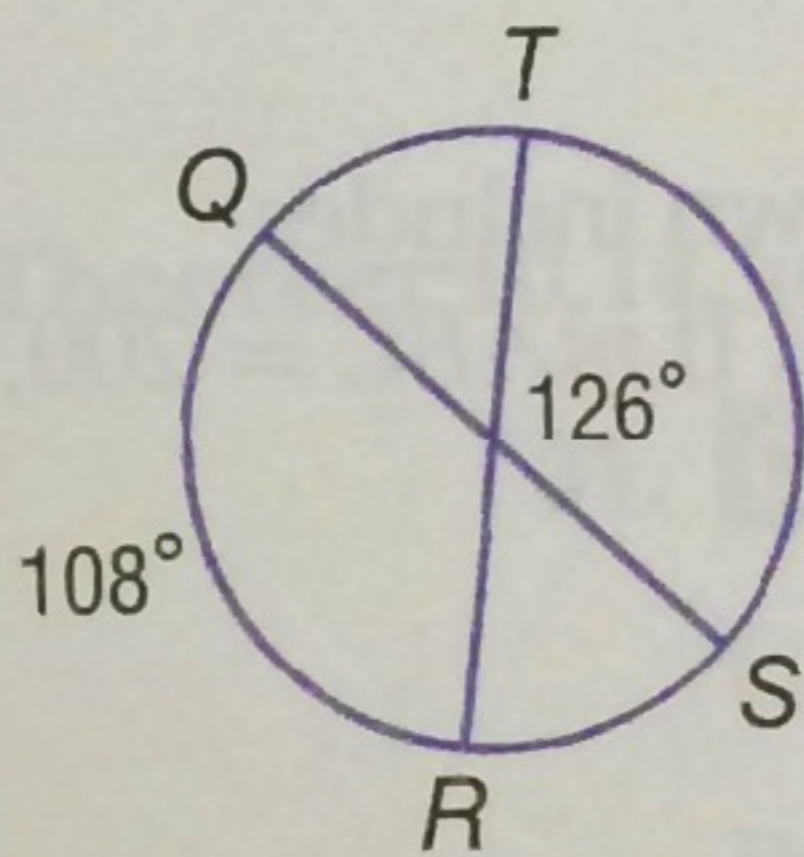


**Examples 1–2** Find each measure. Assume that segments that appear to be tangent are tangent.

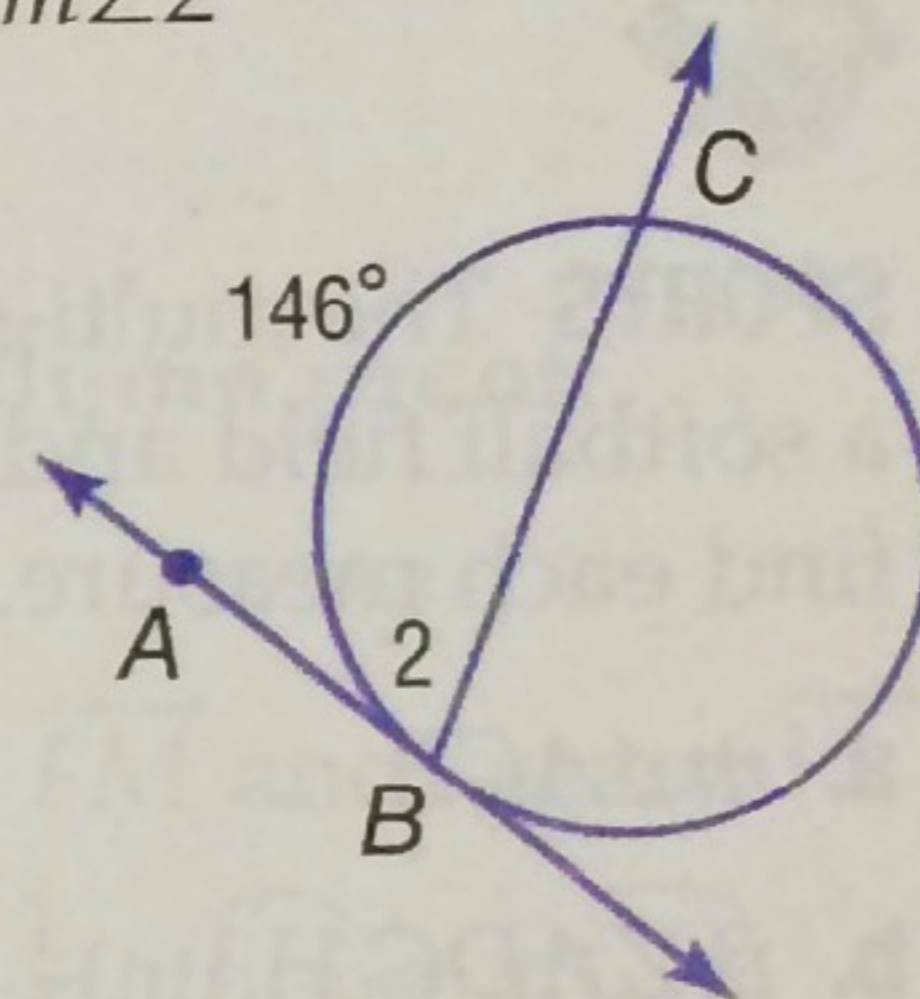
1.  $m\angle 1$



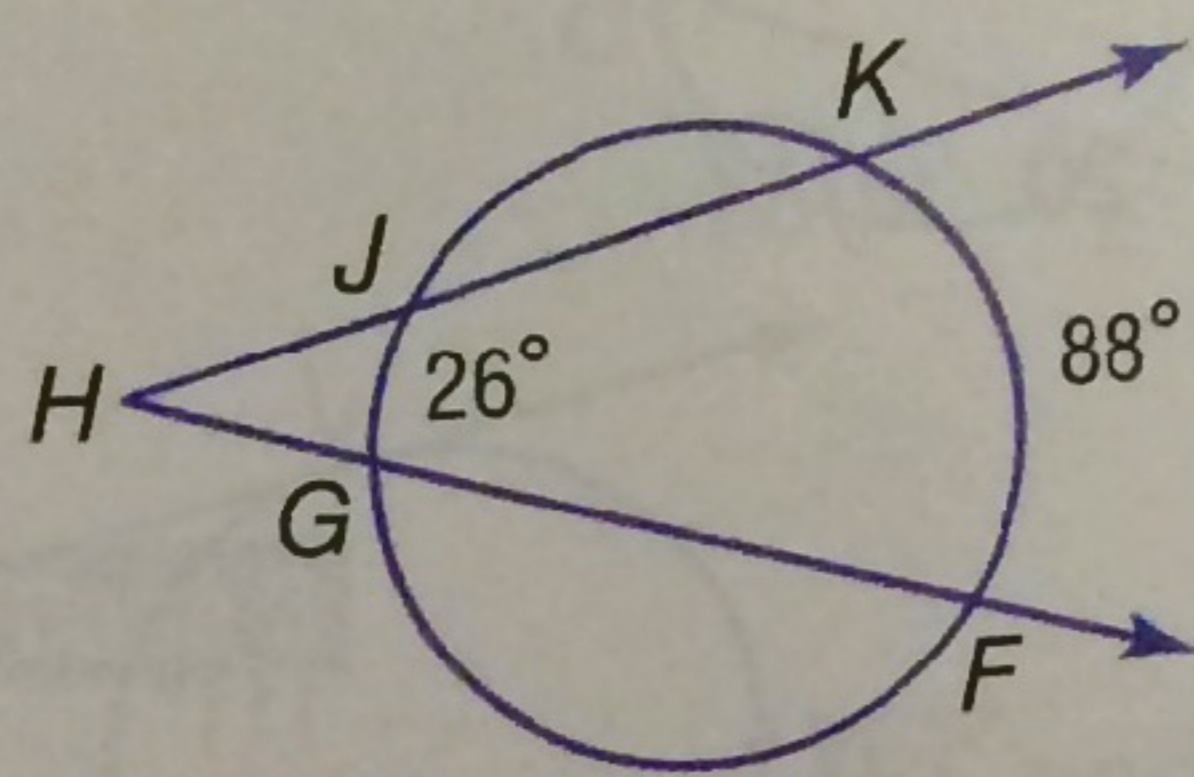
2.  $m\widehat{TS}$



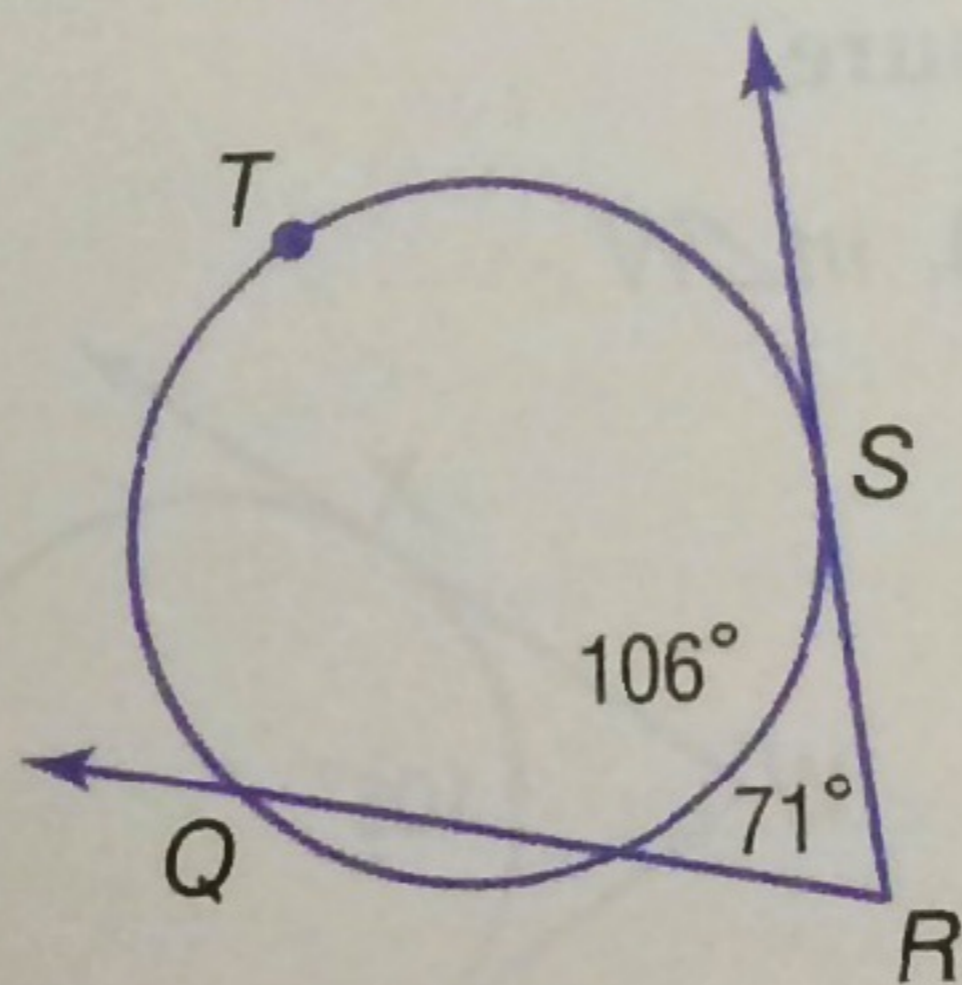
3.  $m\angle 2$



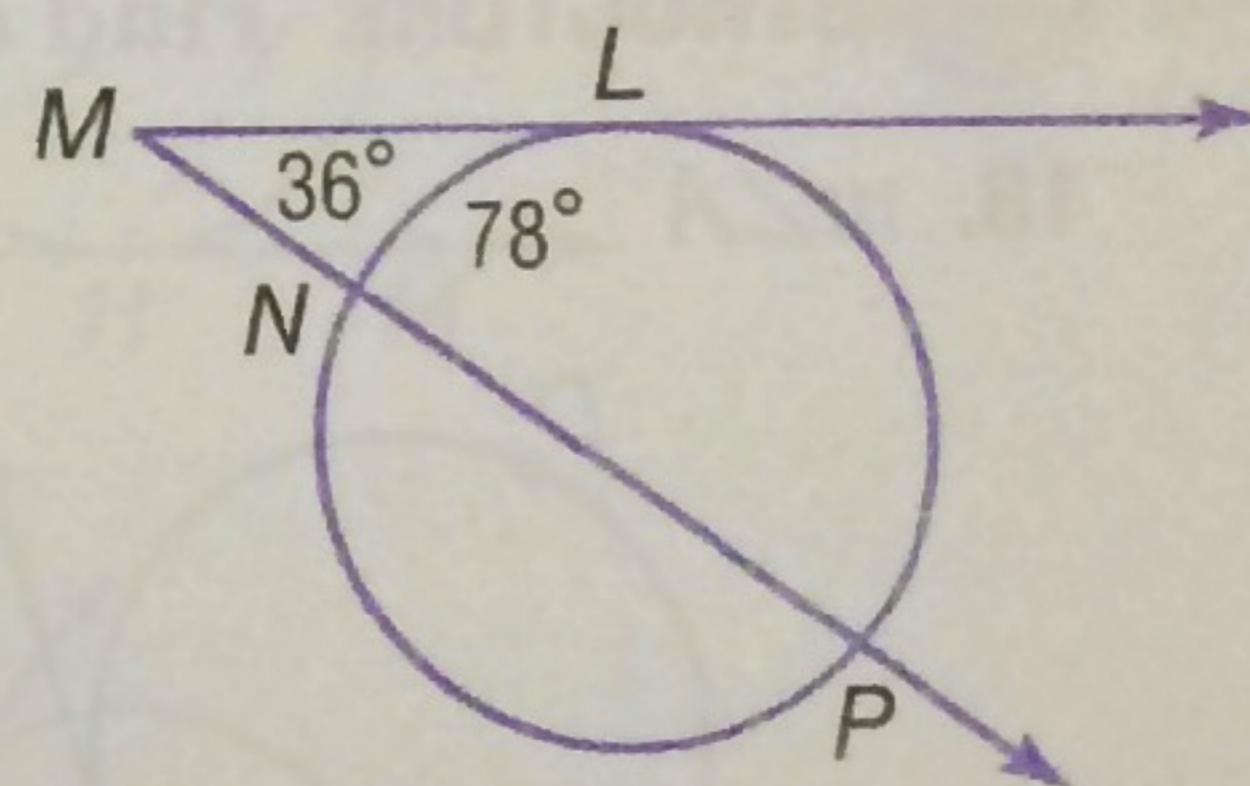
**Examples 3–4** 4.  $m\angle H$



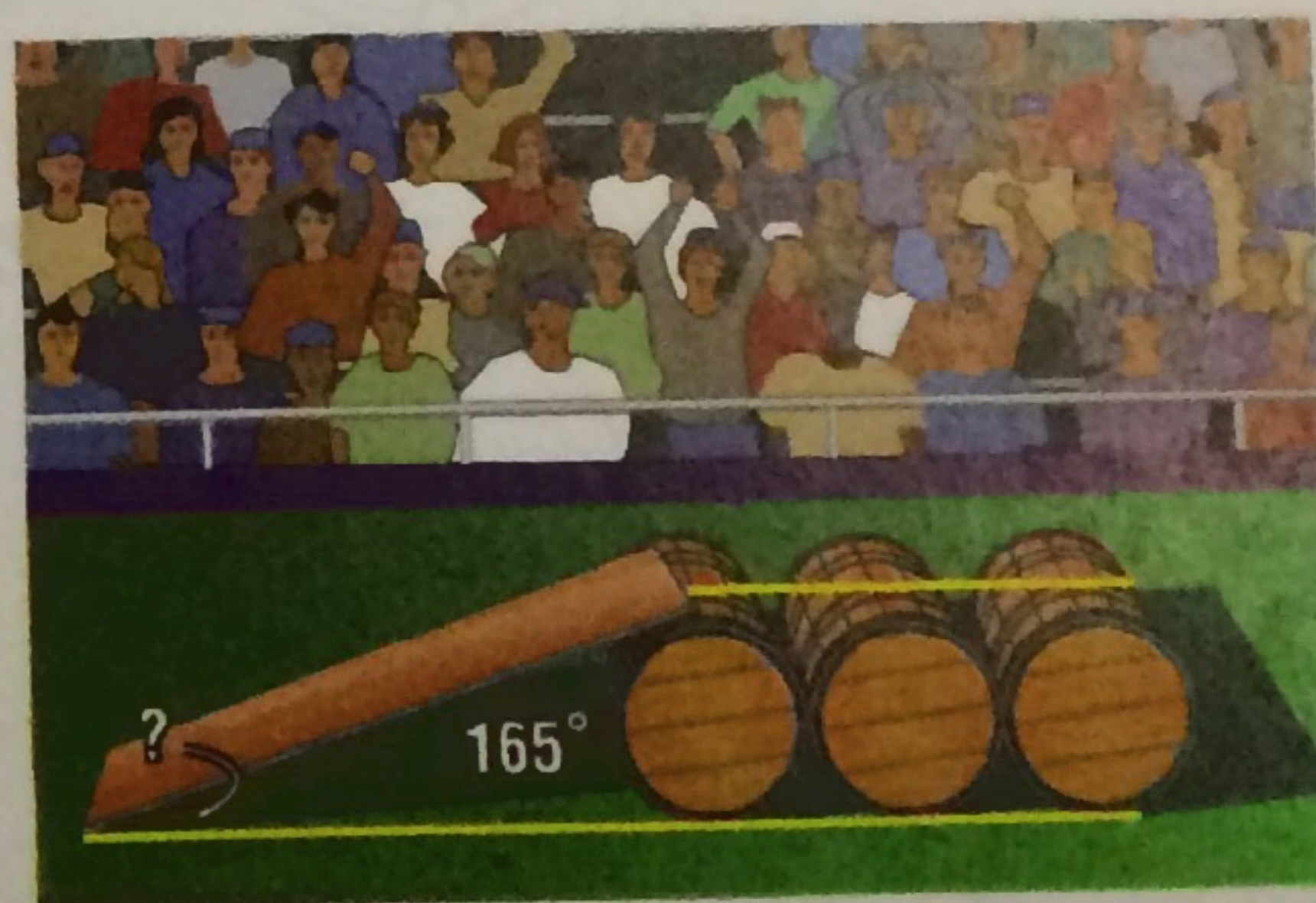
5.  $m\widehat{QTS}$



6.  $m\widehat{LP}$



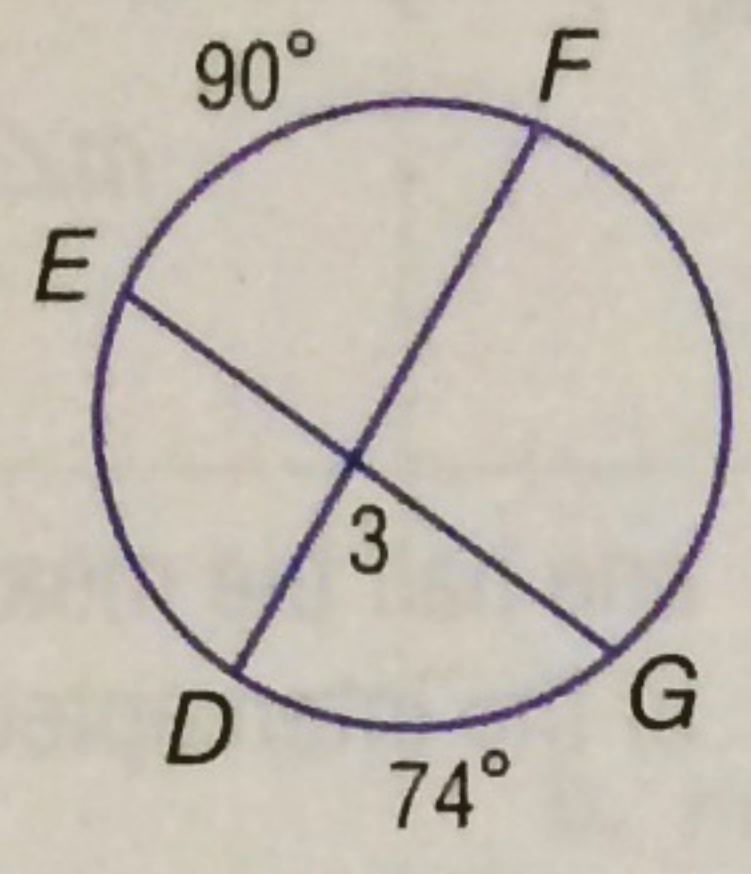
**7 STUNTS** A ramp is attached to the first of several barrels that have been strapped together for a circus motorcycle stunt as shown. What is the measure of the angle the ramp makes with the ground?



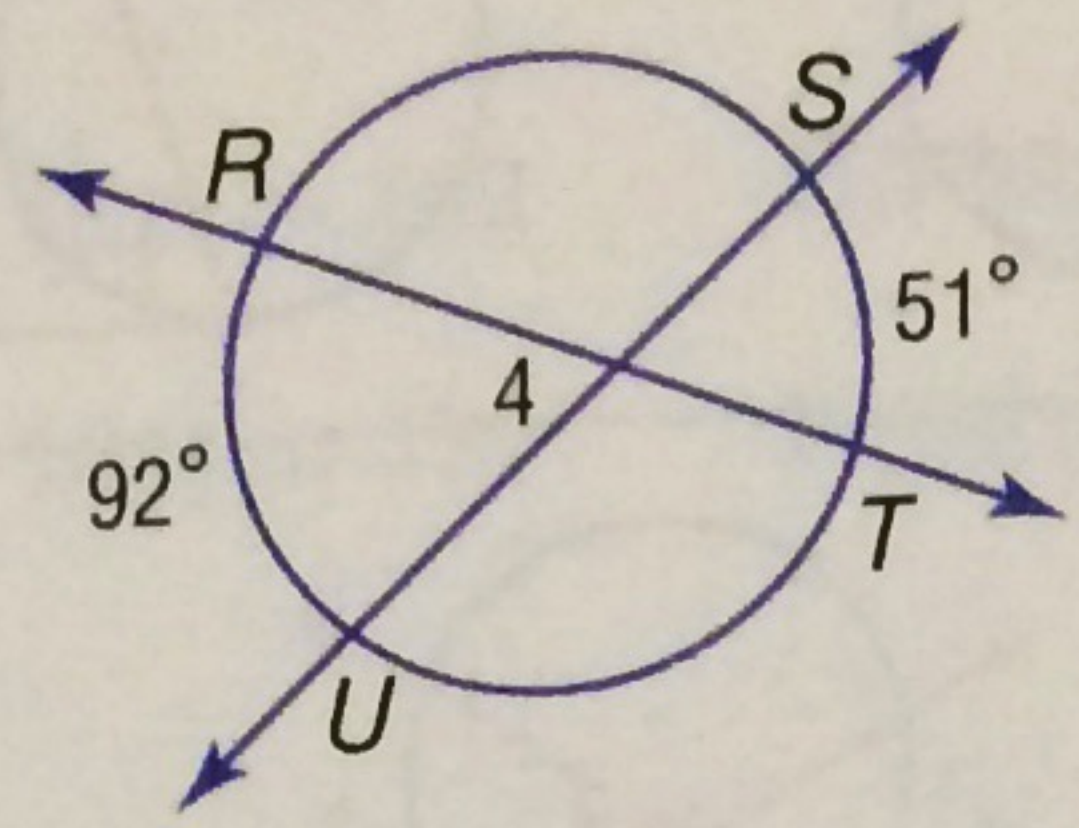
**Practice and Problem Solving**

**Examples 1–2** Find each measure. Assume that segments that appear to be tangent are tangent.

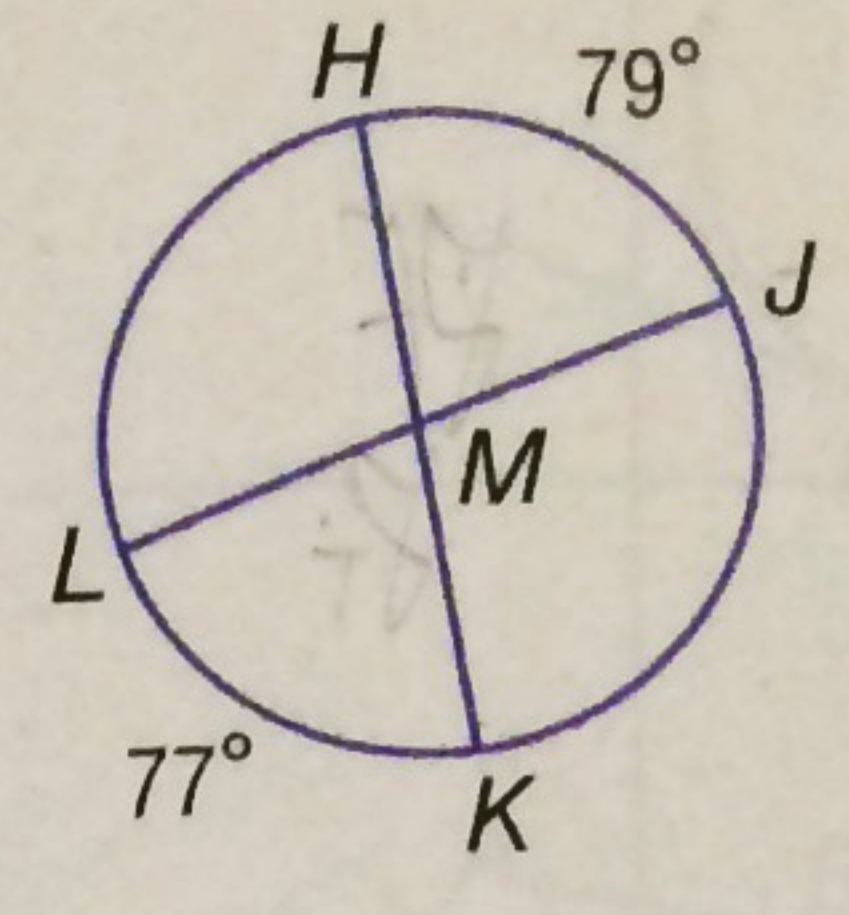
8.  $m\angle 3$



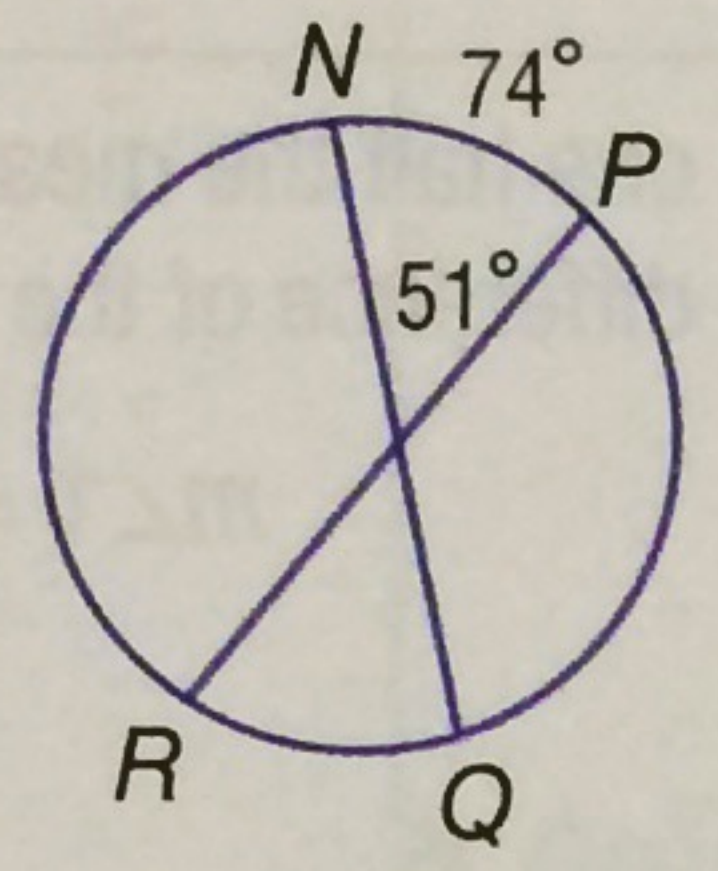
9.  $m\angle 4$



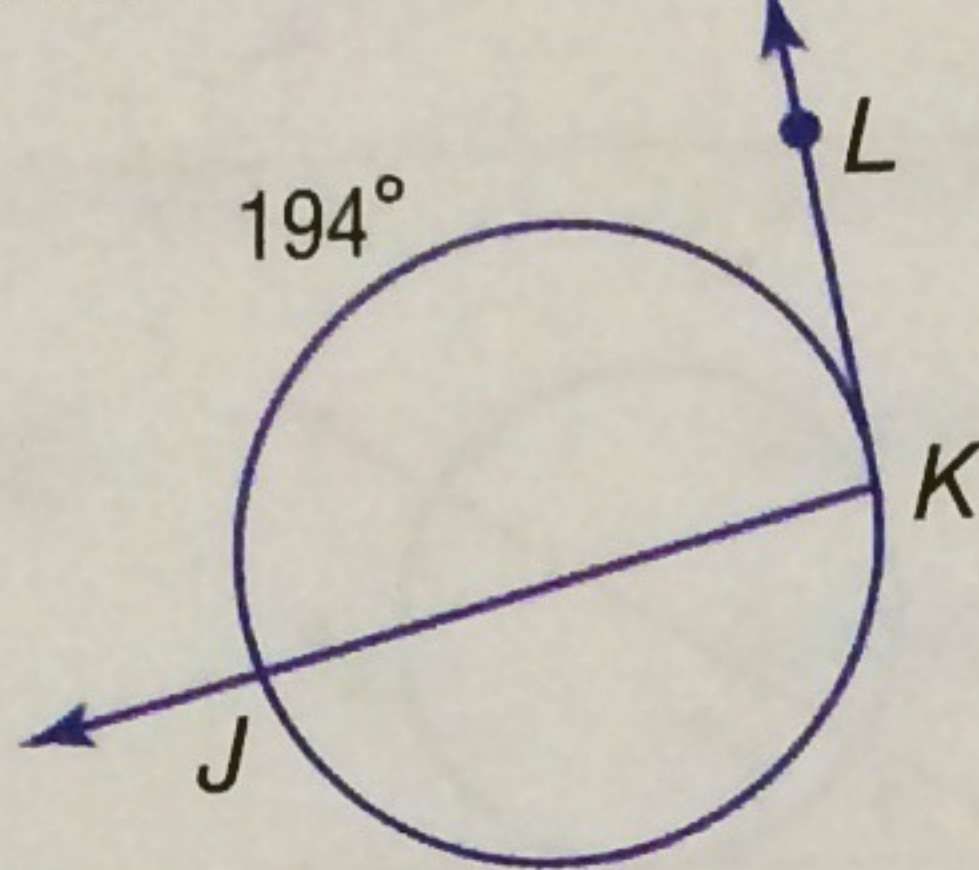
10.  $m\angle JMK$



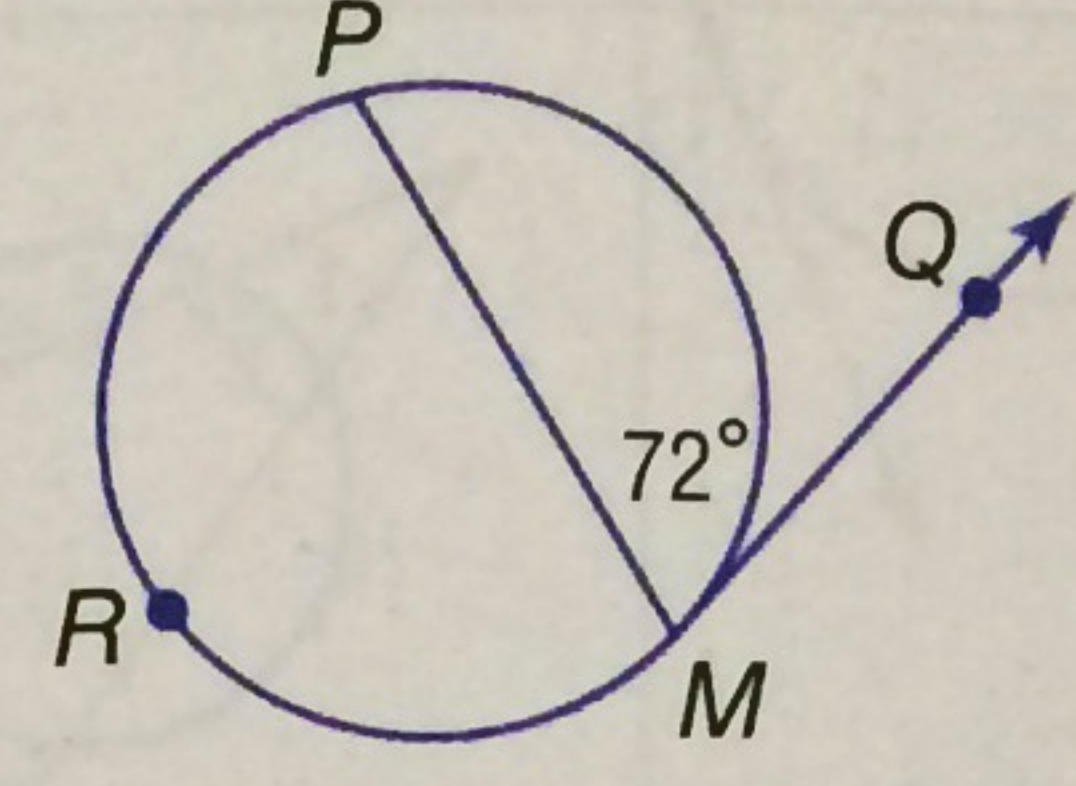
11.  $m\widehat{RQ}$



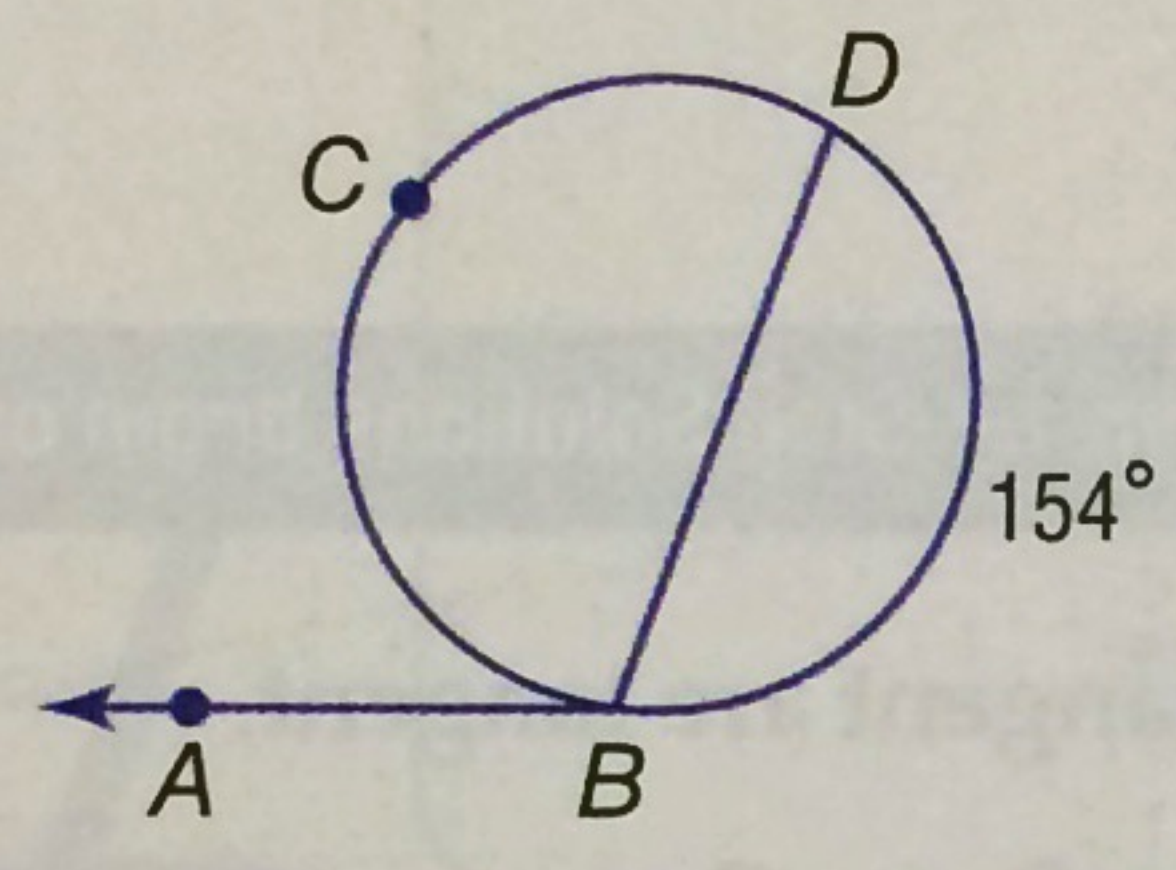
12.  $m\angle K$



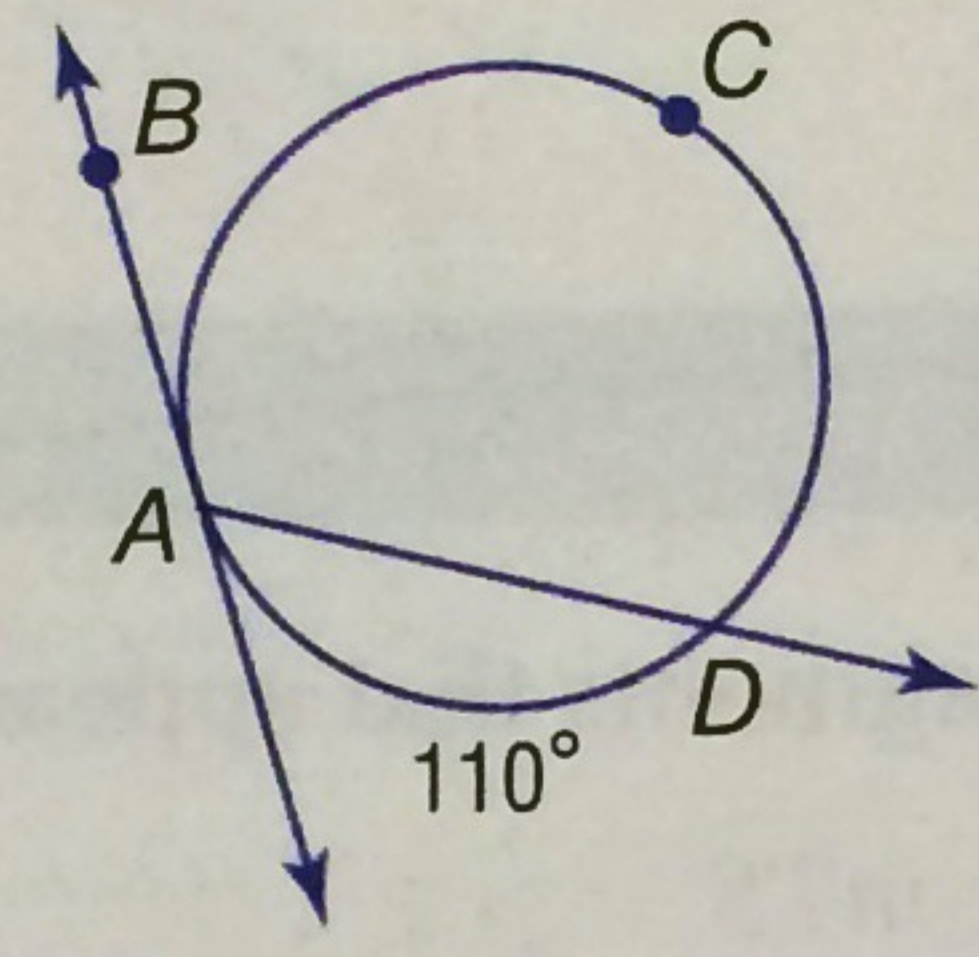
13.  $m\widehat{PM}$



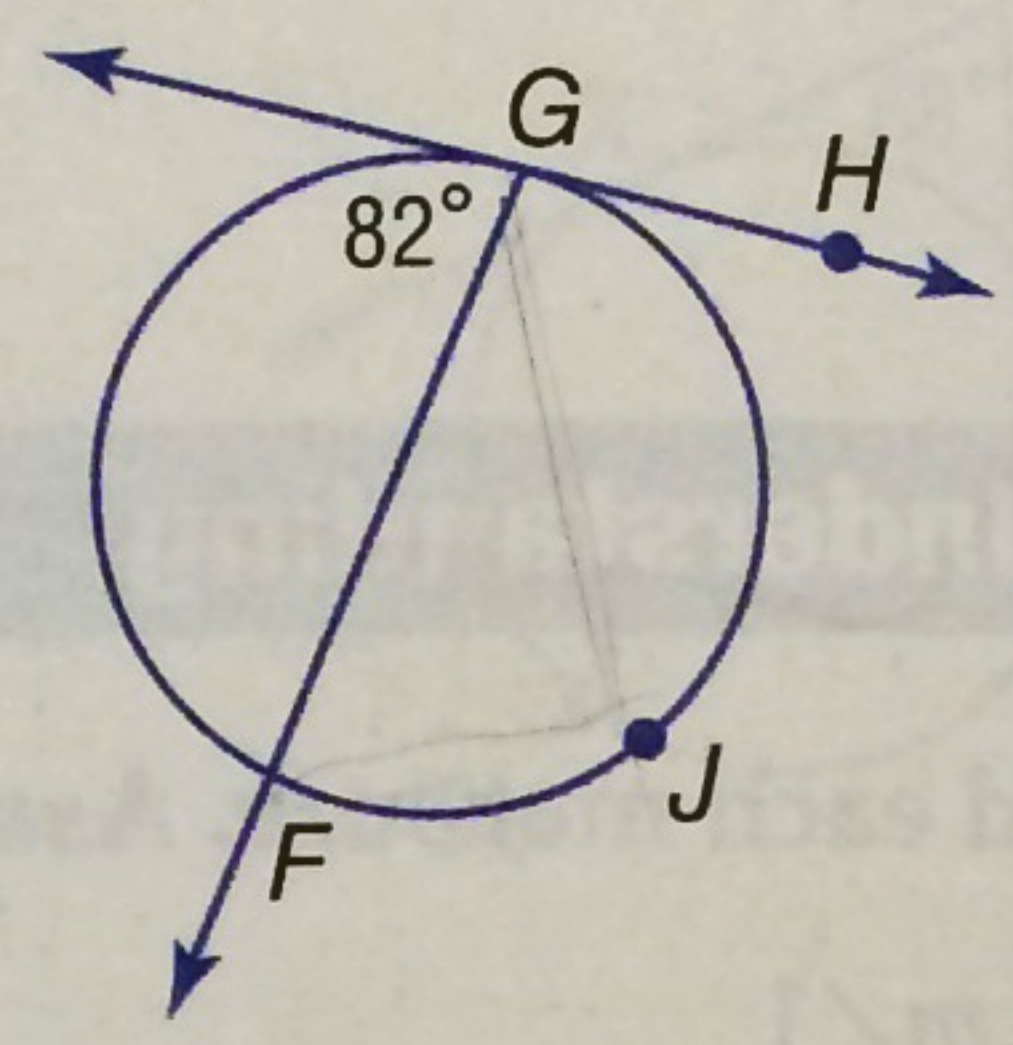
14.  $m\angle ABD$



15.  $m\angle DAB$

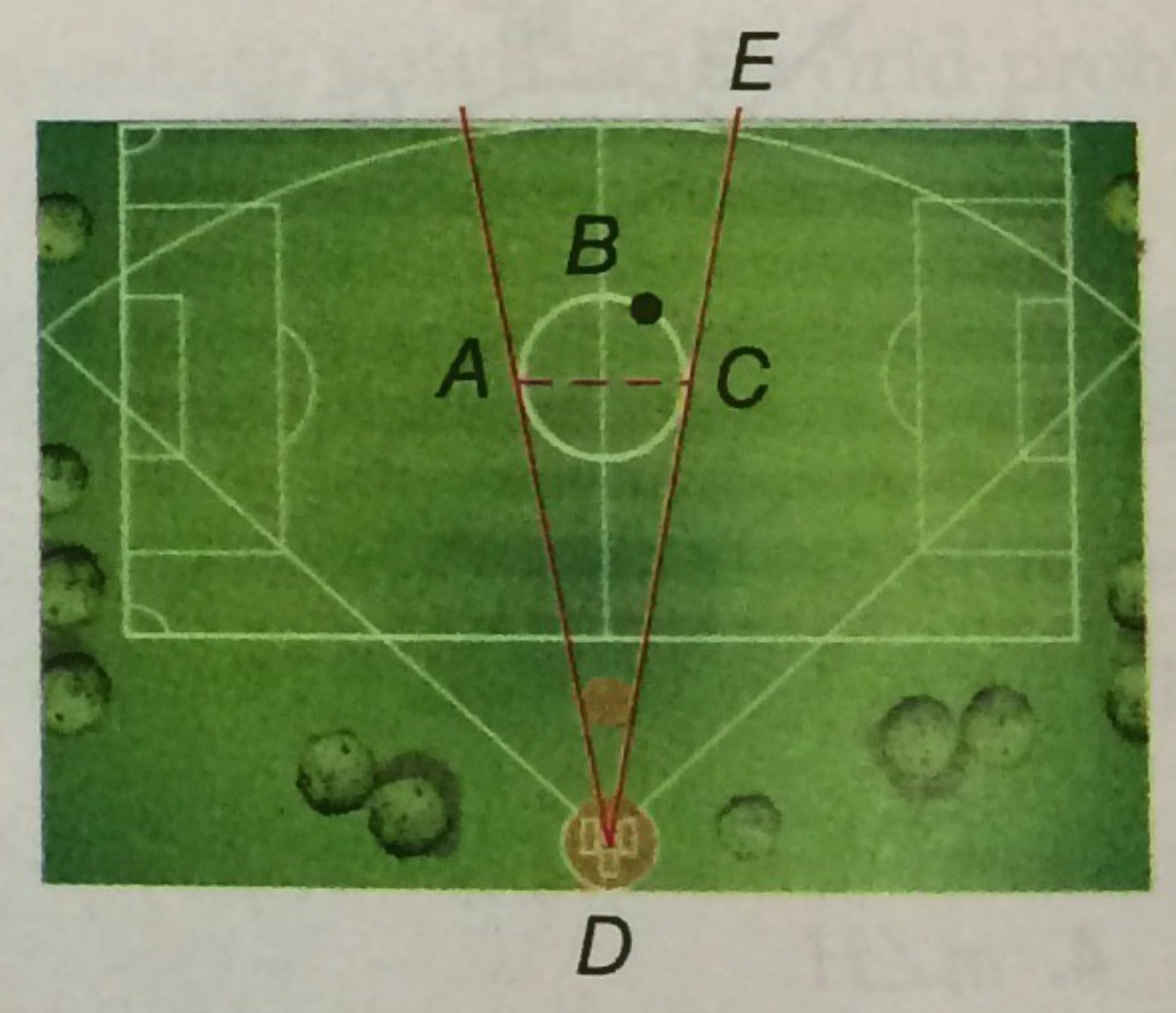


16.  $m\widehat{GJF}$



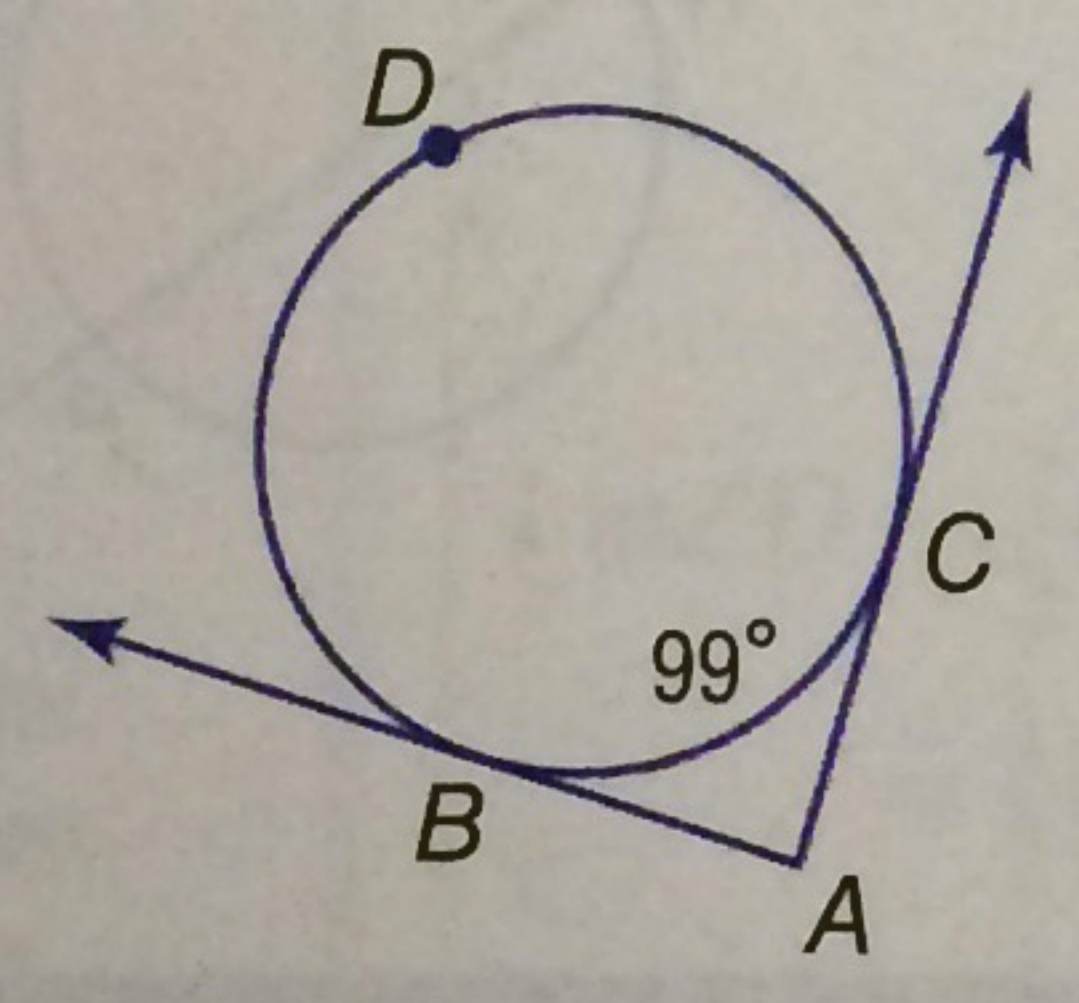
17. **SPORTS** The multi-sport field shown includes a softball field and a soccer field. If  $m\widehat{ABC} = 200$ , find each measure.

- a.  $m\angle ACE$
- b.  $m\angle ADC$

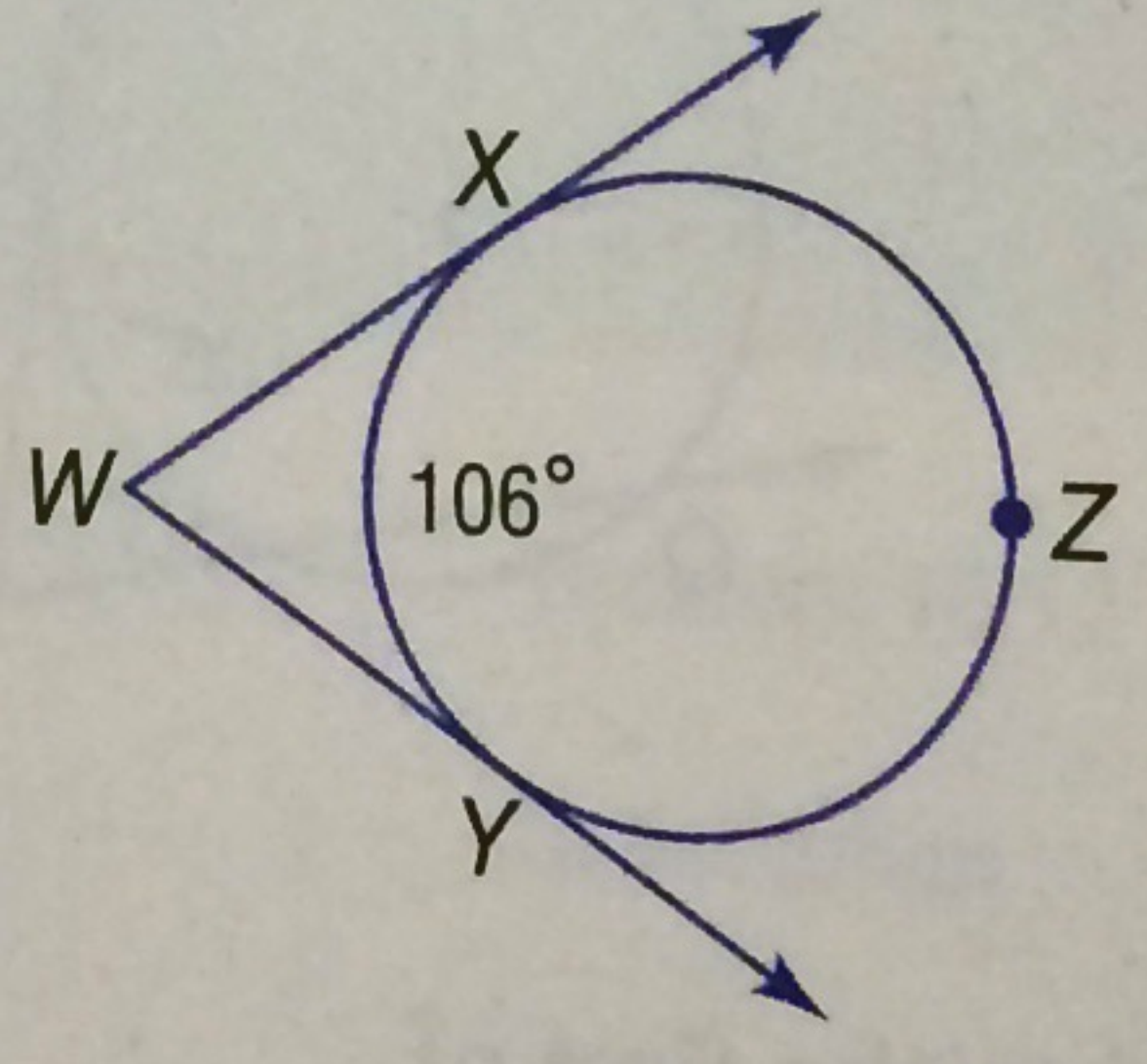


**Examples 3–4** **CCSS STRUCTURE** Find each measure.

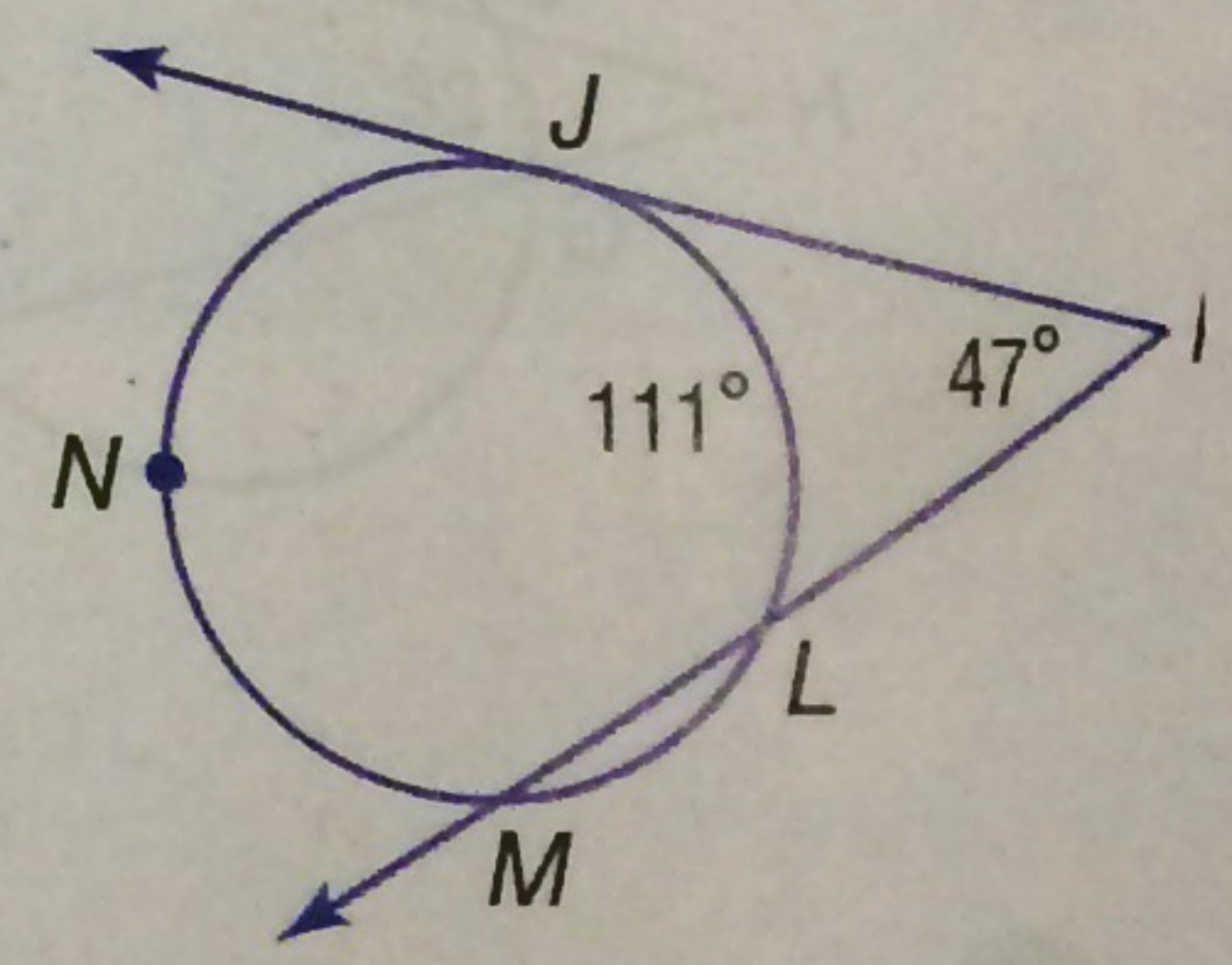
18.  $m\angle A$



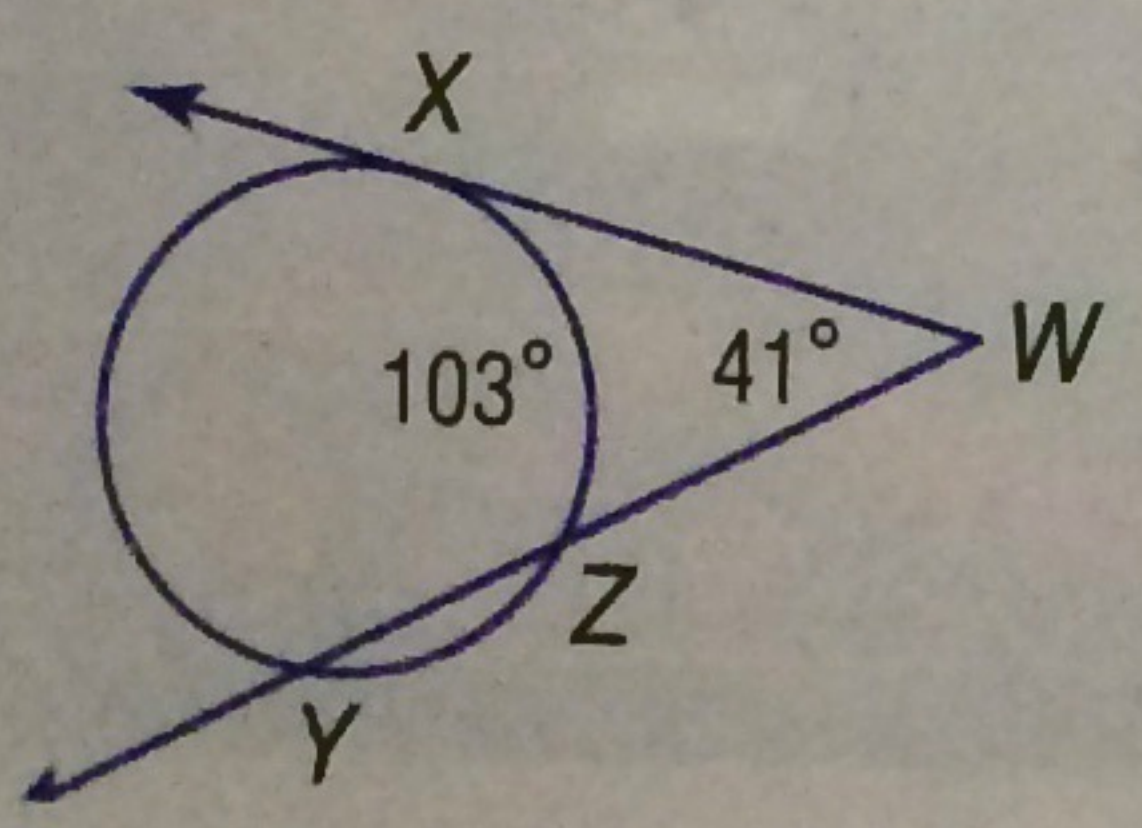
19.  $m\angle W$



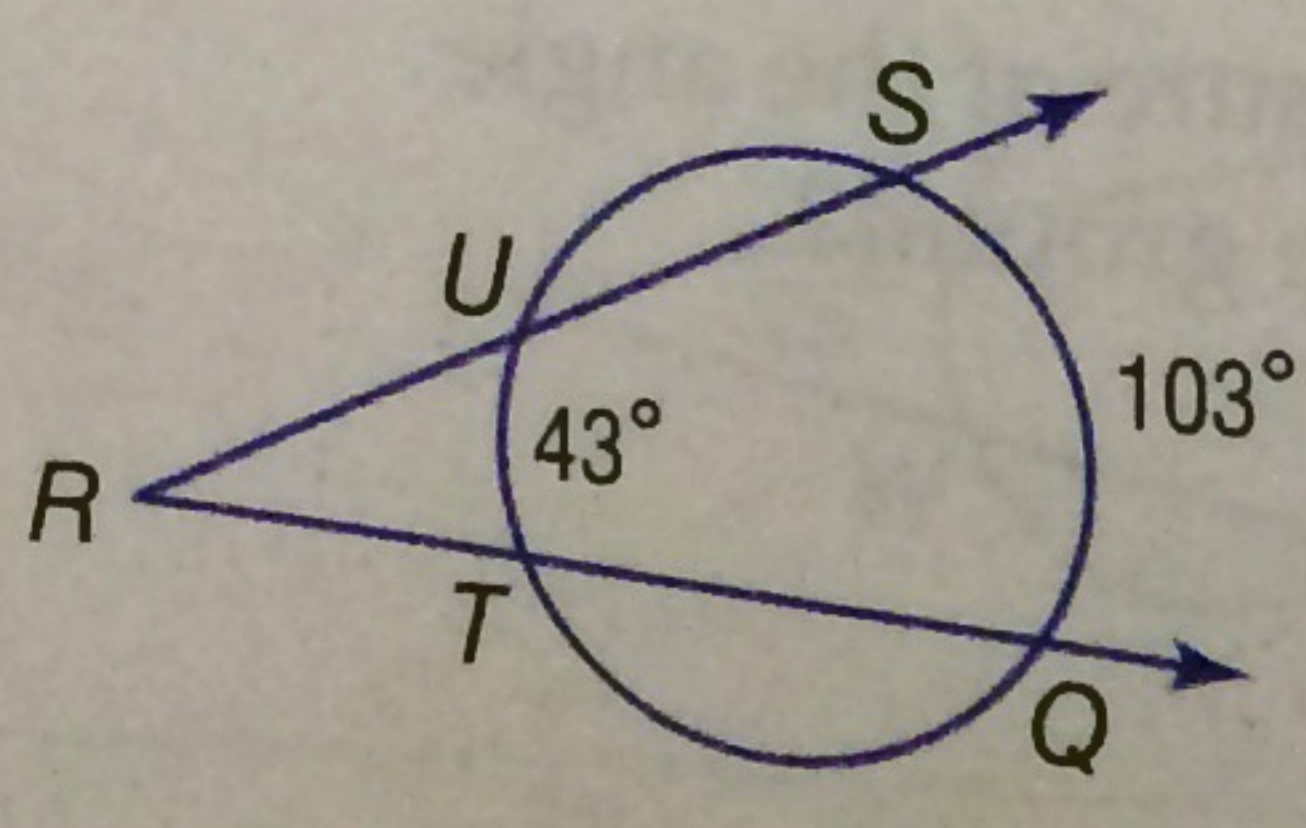
20.  $m\widehat{JM}$



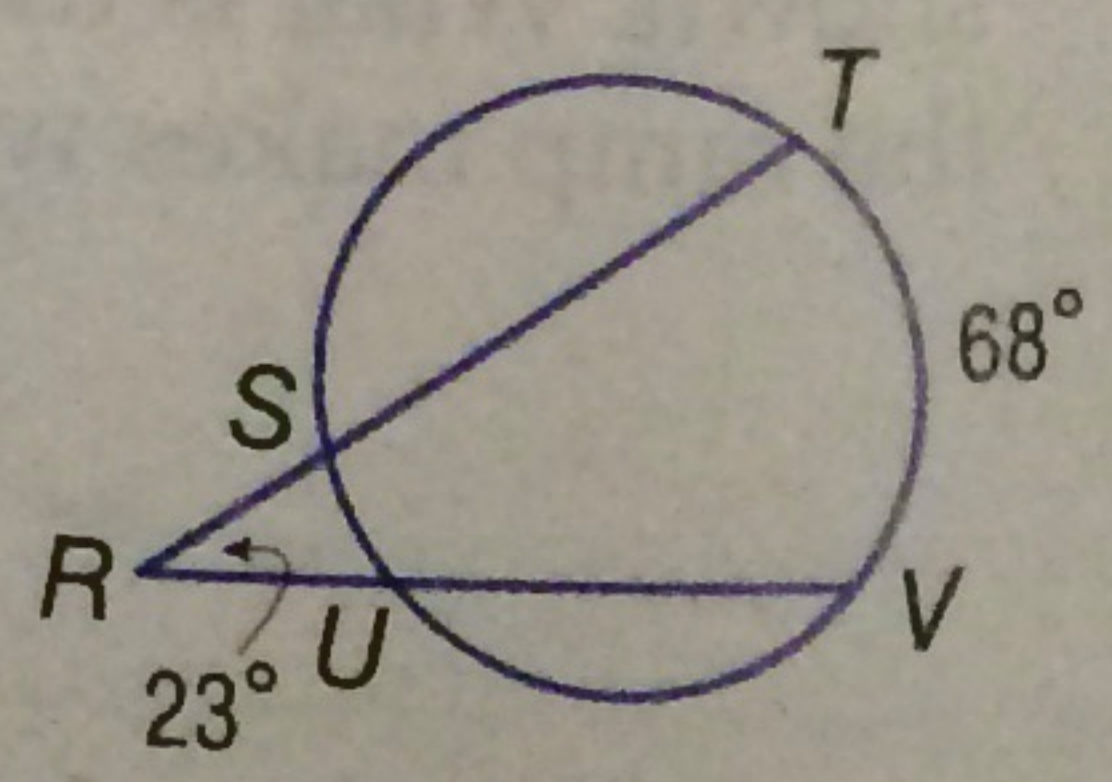
21.  $m\widehat{XY}$



22.  $m\angle R$

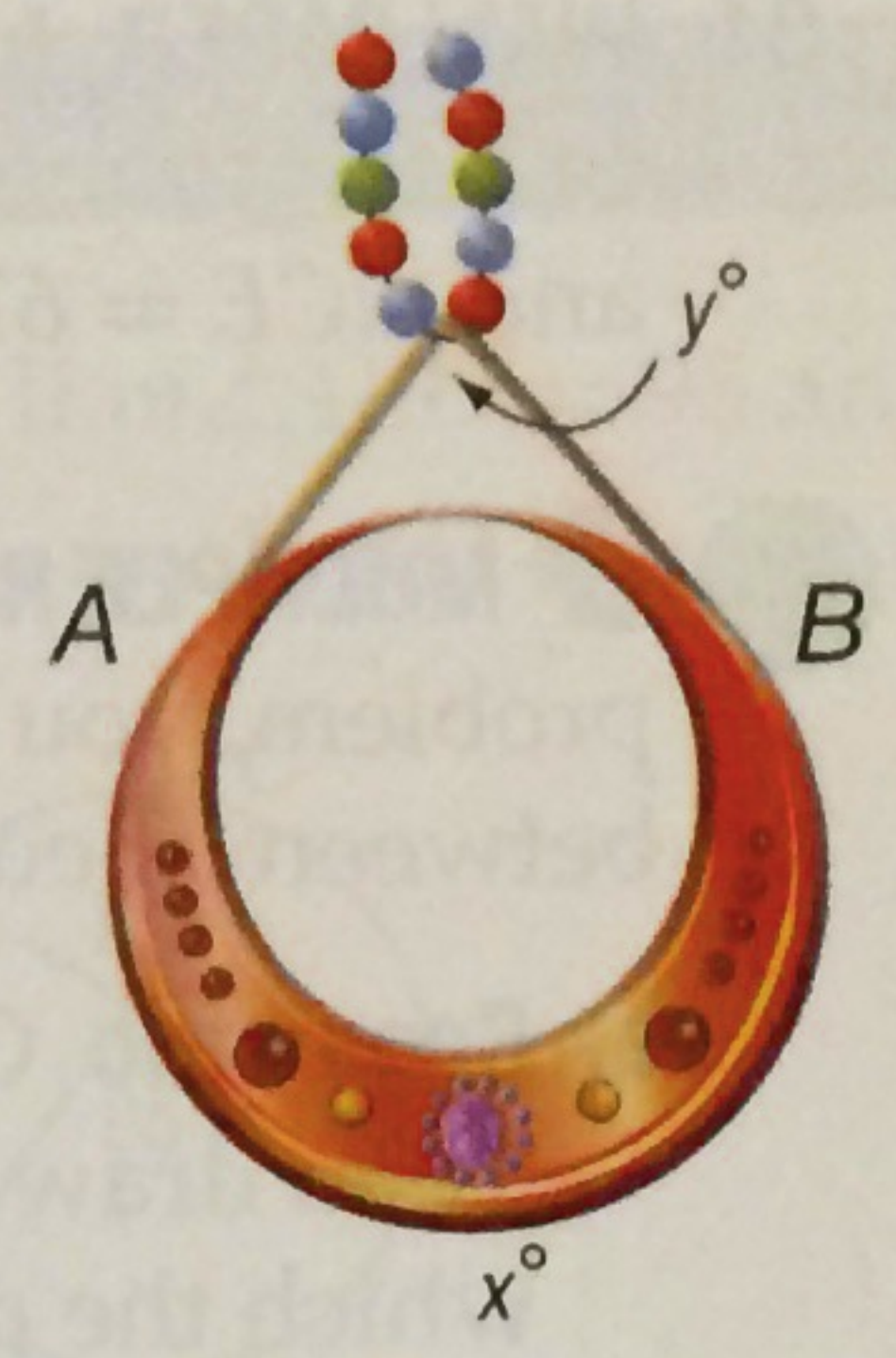
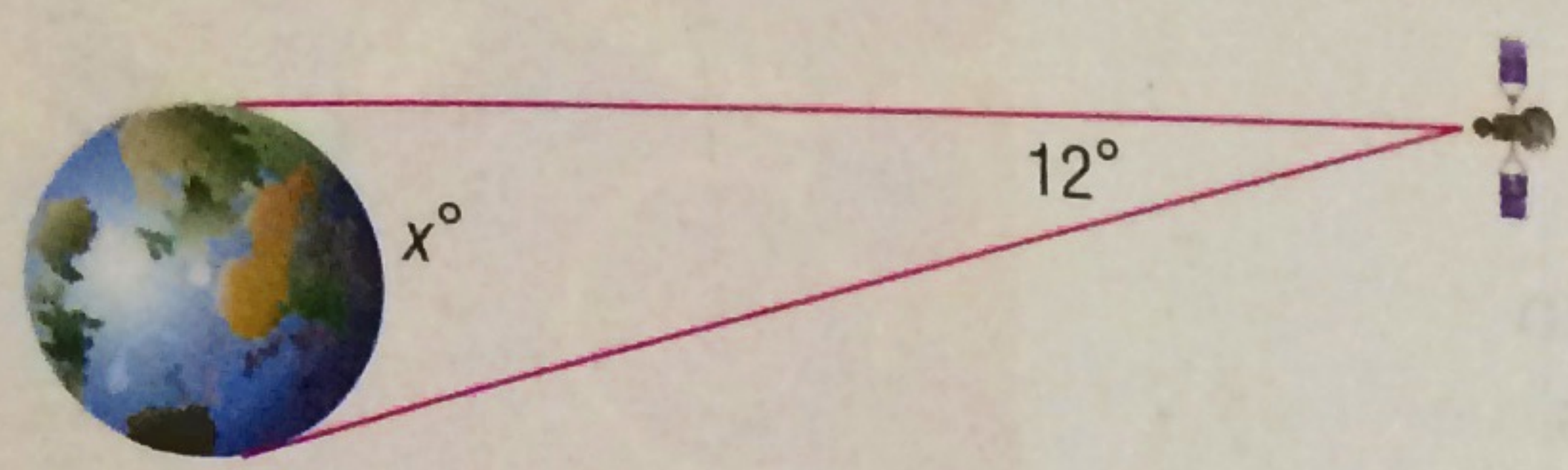


23.  $m\widehat{SU}$

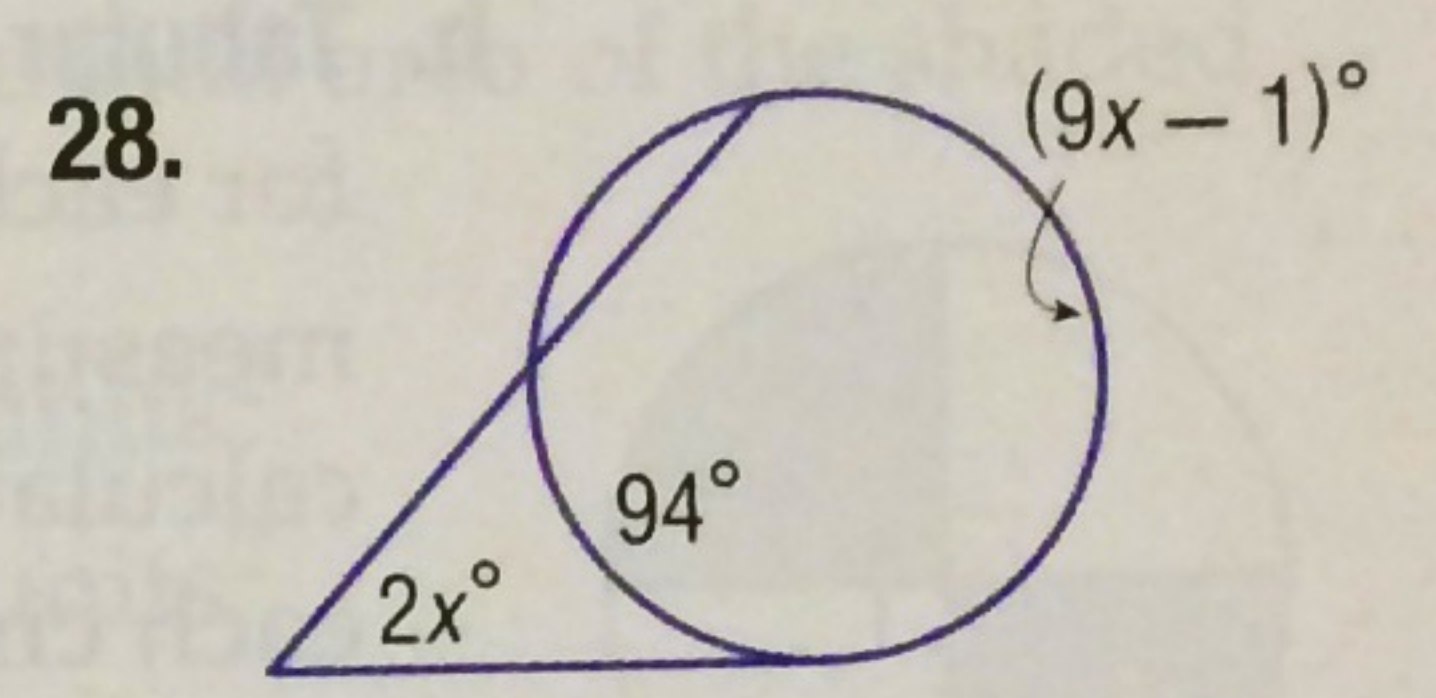
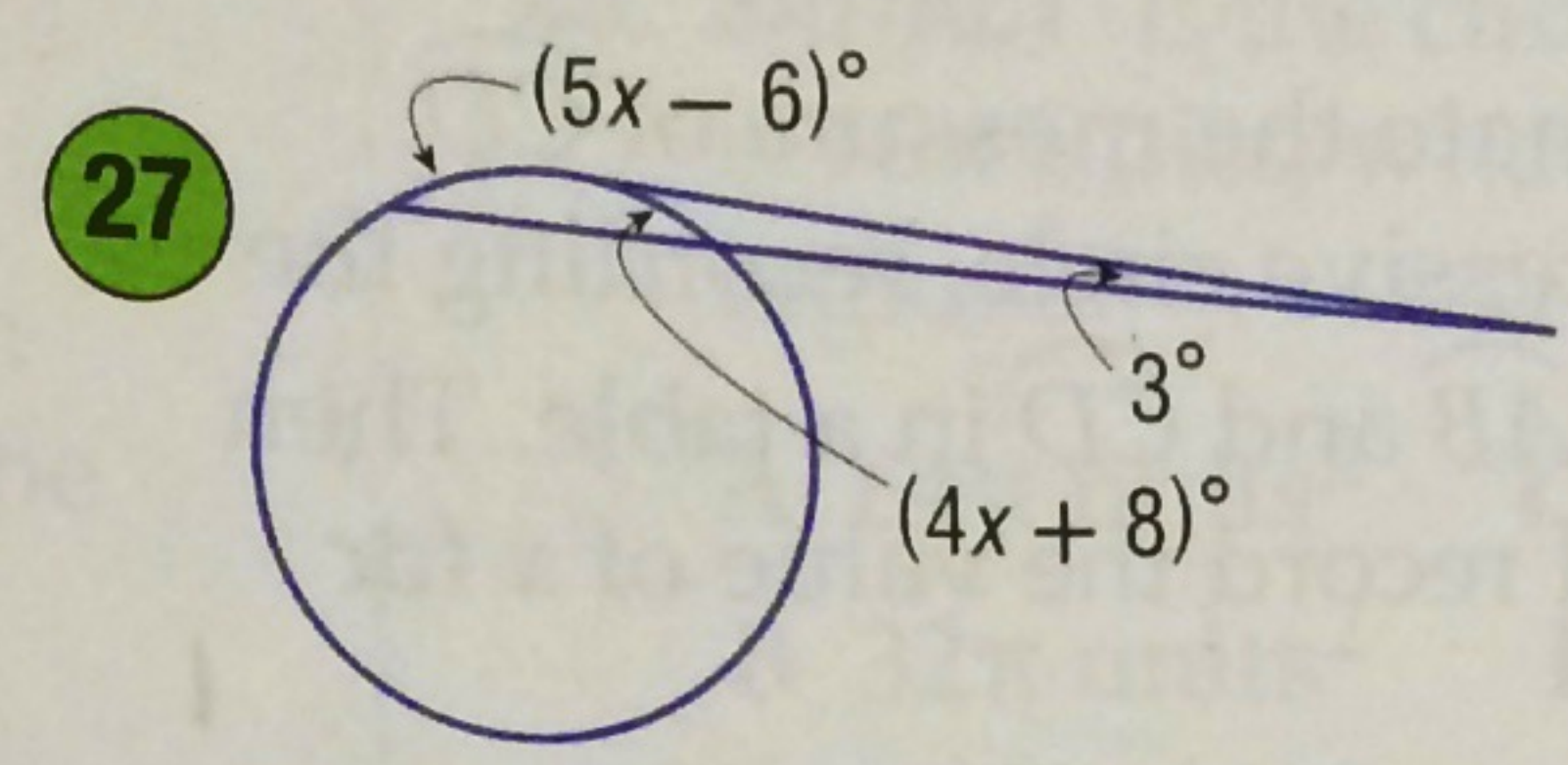
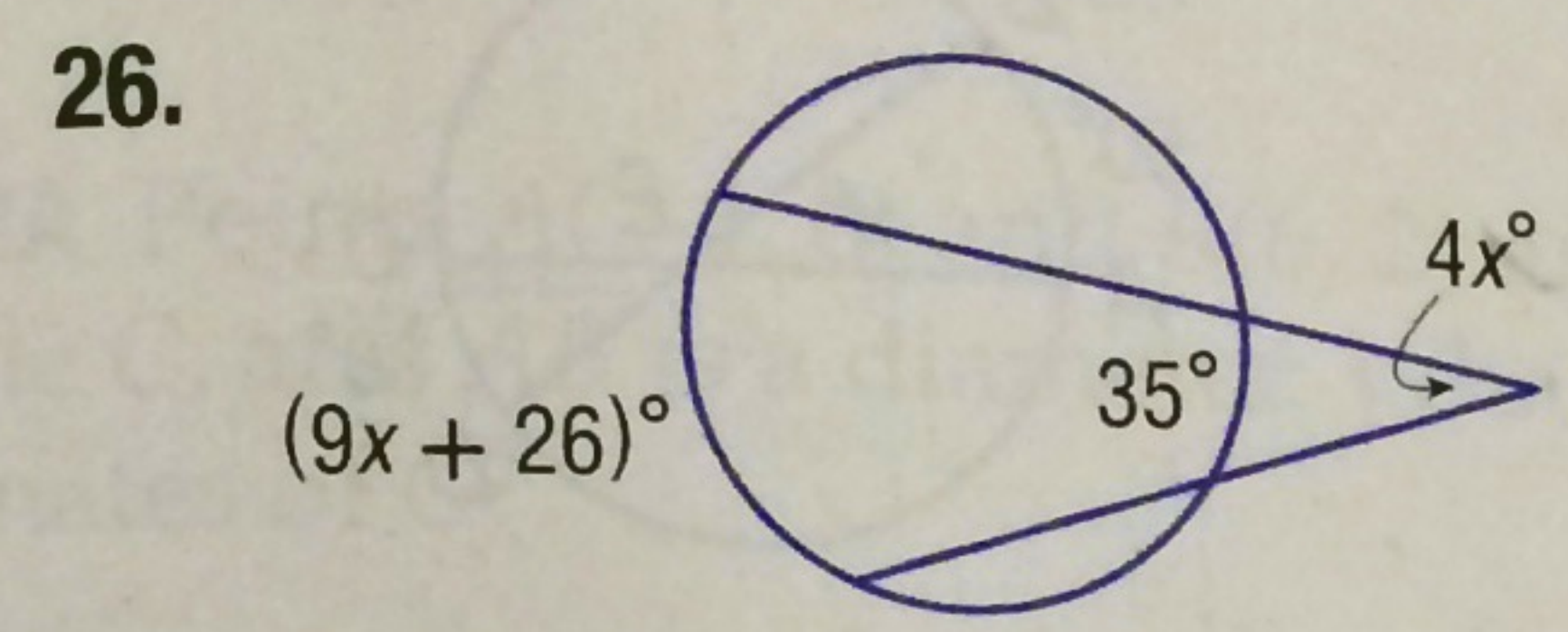


24. **JEWELRY** In the circular necklace shown, A and B are tangent points. If  $x = 260$ , what is  $y$ ?

25. **SPACE** A satellite orbits above Earth's equator. Find  $x$ , the measure of the planet's arc, that is visible to the satellite.

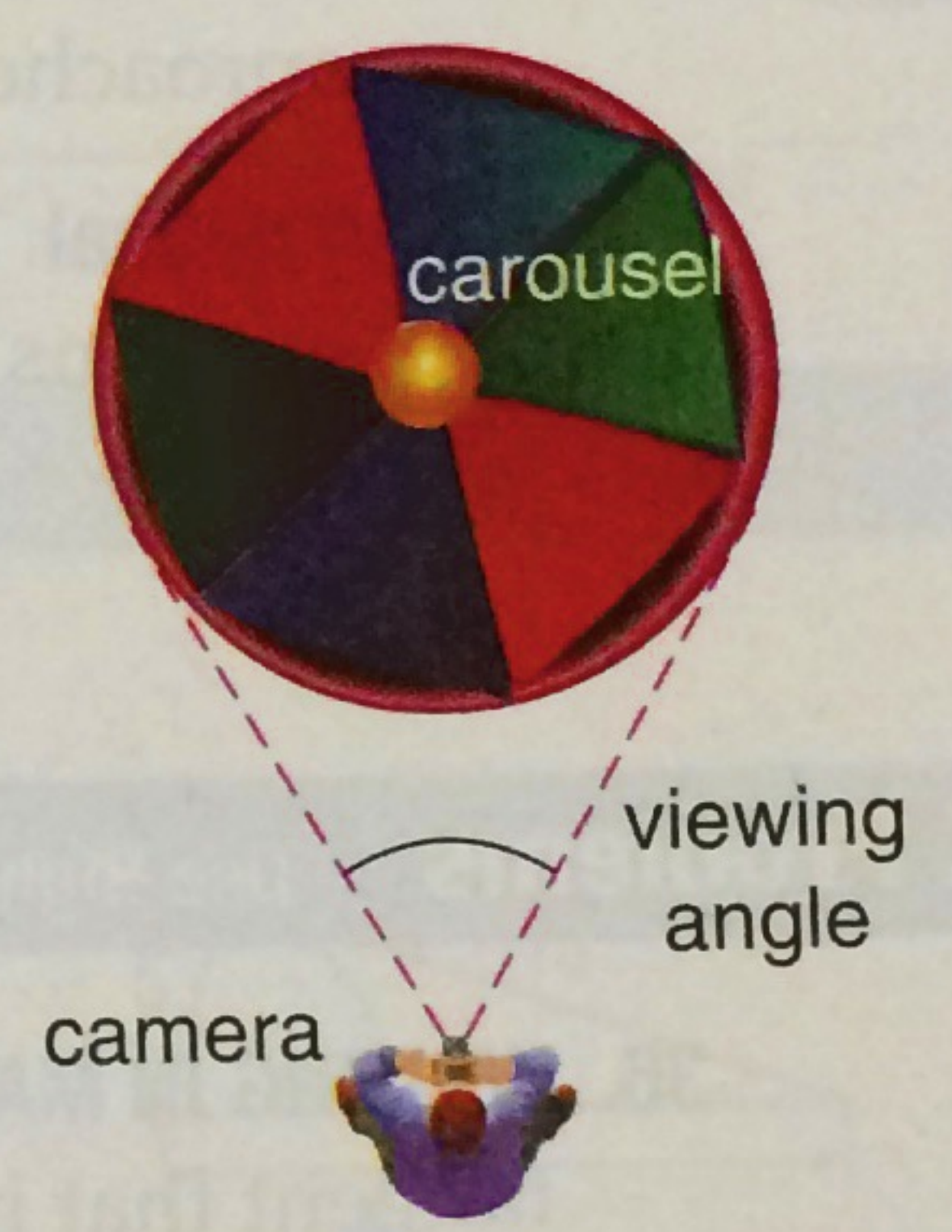


**ALGEBRA** Find the value of  $x$ .



29. **PHOTOGRAPHY** A photographer frames a carousel in his camera shot as shown so that the lines of sight form tangents to the carousel.

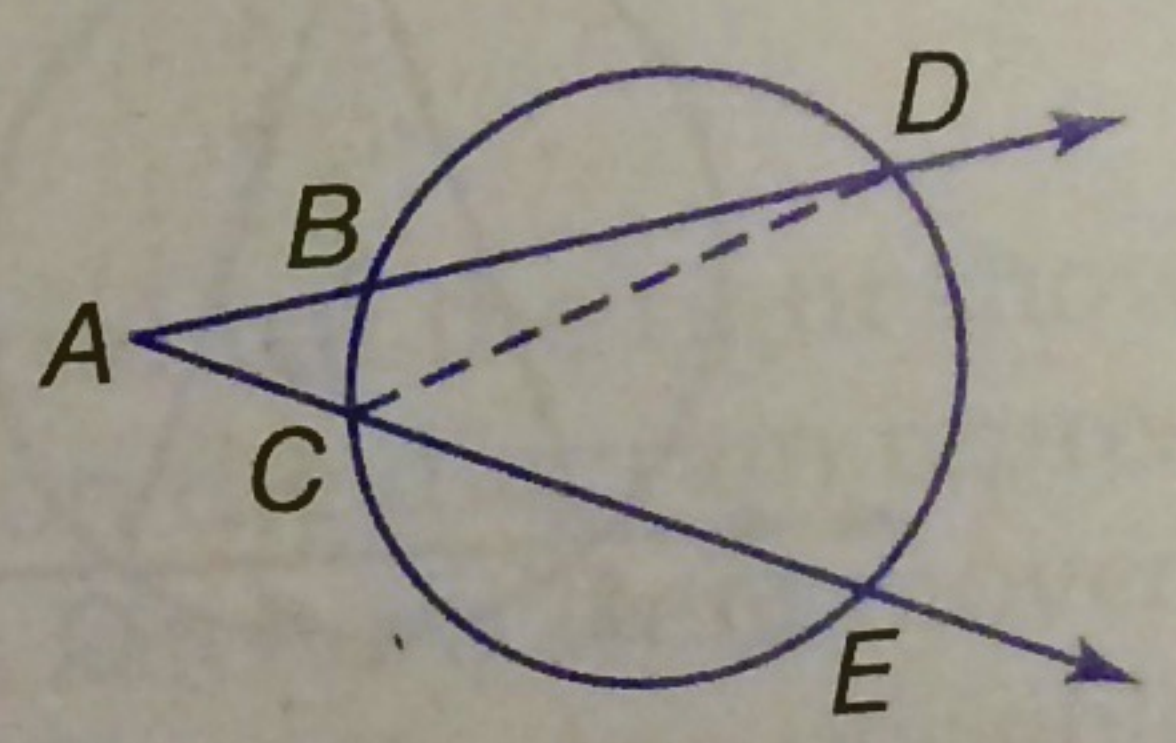
- a. If the camera's viewing angle is  $35^\circ$ , what is the arc measure of the carousel that appears in the shot?
- b. If you want to capture an arc measure of  $150^\circ$  in the photograph, what viewing angle should be used?



**CCSS ARGUMENTS** For each case of Theorem 10.14, write a two-column proof.

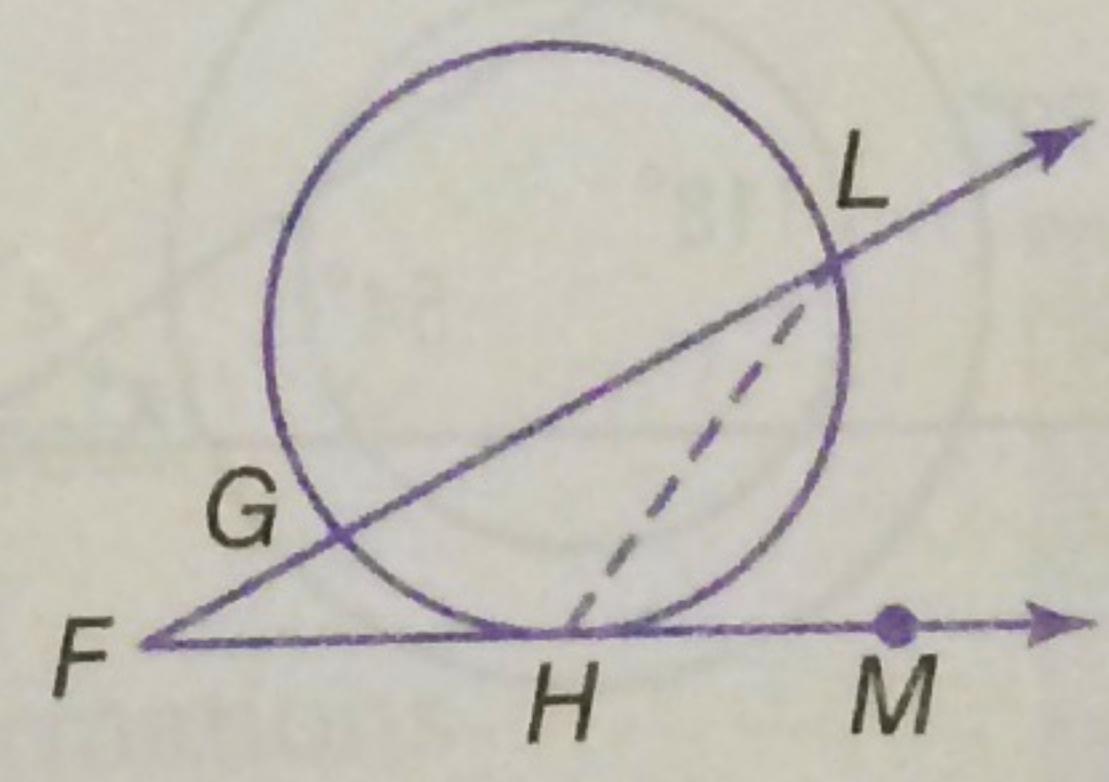
30. **Case 1**

Given: secants  $\overrightarrow{AD}$  and  $\overrightarrow{AE}$   
 Prove:  $m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$



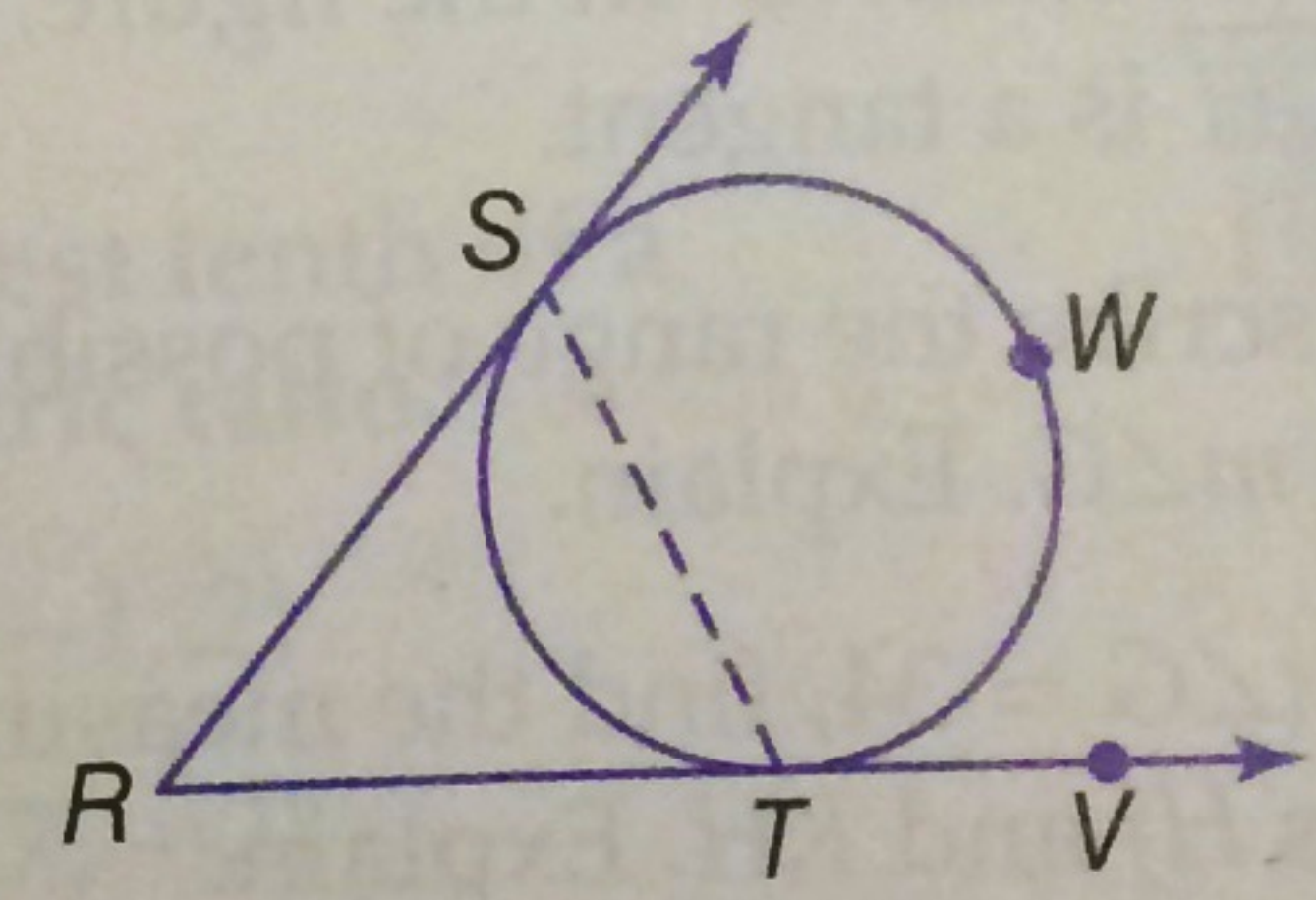
31. **Case 2**

Given: tangent  $\overrightarrow{FM}$  and secant  $\overrightarrow{FL}$   
 Prove:  $m\angle F = \frac{1}{2}(m\widehat{LH} - m\widehat{GH})$



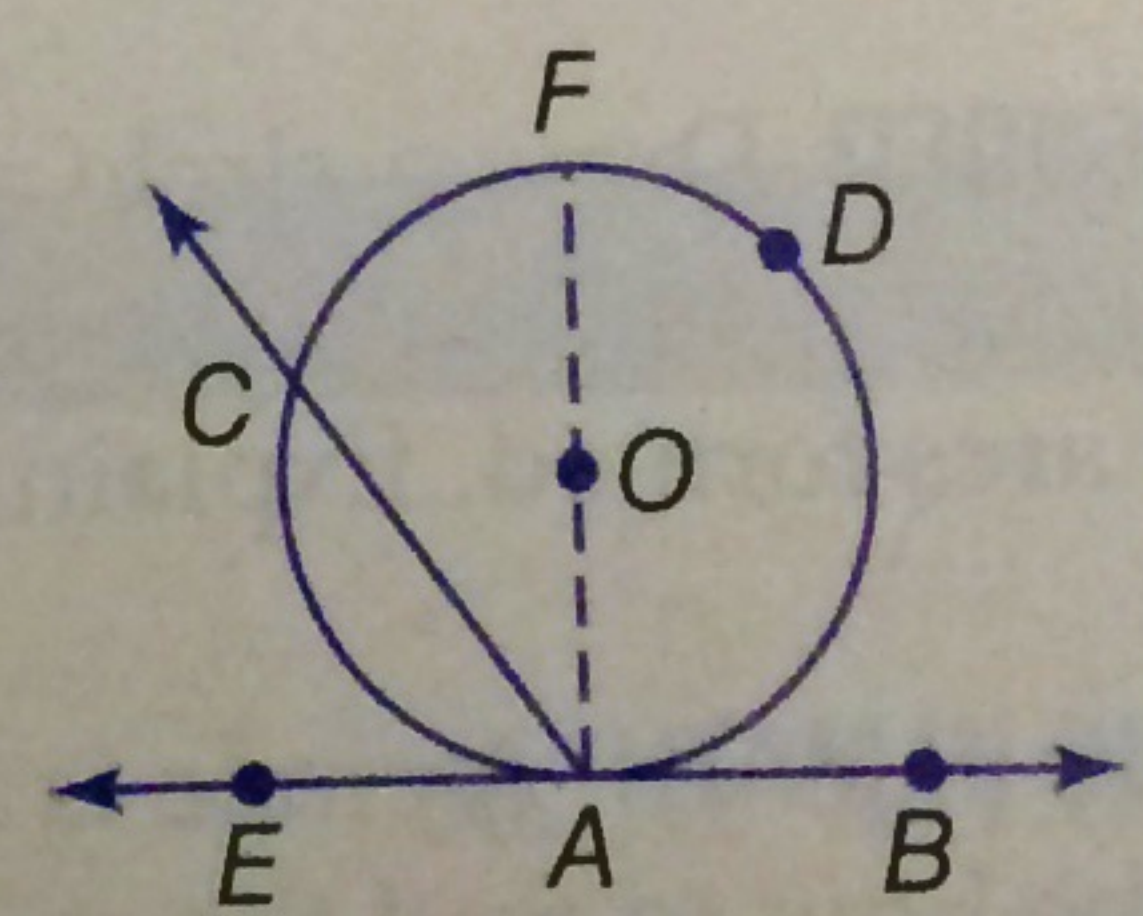
32. **Case 3**

Given: tangents  $\overrightarrow{RS}$  and  $\overrightarrow{RV}$   
 Prove:  $m\angle R = \frac{1}{2}(m\widehat{SWT} - m\widehat{ST})$



33. **PROOF** Write a paragraph proof of Theorem 10.13.

- a. Given:  $\overrightarrow{AB}$  is a tangent of  $\odot O$ .
- $\overrightarrow{AC}$  is a secant of  $\odot O$ .
- $\angle CAE$  is acute.



$m\angle CAE = \frac{1}{2}m\widehat{CA}$